



# Study Guide

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Educ8 tuition centre's would like to thank you for your support.

We have spent over a year designing this product which we feel will improve students' understanding in mathematics at the GR 12 and GR 11 level.

This product is not meant to be used as a substitute for work covered at schools but rather as a compliment to work covered at school.

Onus is on the student to ensure that they follow the instructions carefully and do all of the prescribed work before progressing on with further tutorials.

Whilst every effort has been made to ensure the accuracy of this product, there may still be errors as it is only in its' pilot phase, we apologize for any inaccuracies but accept no liability.

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**Exponents, logs and Surds**  
**Exponents**

*Exponents 1, practice examples 1:*

Break the following numbers into their simplest surd form:

- 1) 96
- 2) 121
- 3) 798
- 4) 81
- 5) 1024

*Exponents 1, Practice examples 2:*

Simplify:

1)  $8^4 \cdot 16^3$

2)  $\frac{11x^3y^{-4}}{121x^2}$

3)  $(5^6)^{\frac{1}{18}}$

4)  $\frac{(xy)^2}{x^2y}$

5)  $\frac{2^8}{4.2^6}$

Solve for x:

1)  $2^2 \cdot x = 2^2$

2)  $4x^3 = 32$

3)  $\sqrt{6^2 + 3^2} = \sqrt[3]{45}$

4)  $3^0 + 3^x = 28$

5)  $x^{\frac{4}{7}} = 5^3$

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*Exponents 1, exercise:*

Simplify the following:

1)  $\frac{x^{-3}}{x^2 y^{-5}}$

2)  $(8x^6 y^{-5})^{-3}$

3)  $\sqrt[3]{\frac{27x^9}{64y^{12}}}$

4)  $\frac{72x^{5+n}}{108x^n}$

5)  $\frac{x^{n-1} \cdot x^{2n+3}}{(x^{n-1})^2}$

6)  $2x^6 \cdot 8x^4$

7)  $\frac{3 \cdot 2^{x+1} + 2^{x+5}}{3 \cdot 2^{x+2} - 2^x}$

8)  $\frac{3\sqrt{21} \cdot 2\sqrt{10}}{7\sqrt{2} \cdot \sqrt{105}}$

9)  $\frac{3^{x-3} \cdot 27^{x+5}}{81^{-x-4} \cdot 9^{-2x+2}}$

10)  $x^{\frac{1}{3}} \times \sqrt[6]{x^3} \times \frac{1}{\sqrt[3]{x}} \times \sqrt[3]{\frac{1}{x^4}}$

Solve for x:

1)  $3^{-x} = 81$

2)  $19^x = 1$

3)  $\sqrt{5^2 - 3^2} = \sqrt[3]{64}$

4)  $x^{\frac{9}{4}} = 81$

5)  $3^{6x-5} = 81^{7-4x}$

6)  $(3^{-x} + 3)(3^x - 81) = 0$

7)  $x + \sqrt{7-x} + 2 = 0$

8)  $2^{x+1} - 2^{x+4} = -(2^2) - 2^{x+1} - 2^{x+2}$

9)  $5x^{\frac{3}{5}} = 150$

10)  $\frac{(3^x)^{2x} \cdot \sqrt{9^{-3}}}{3^x} = 1$

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**Exponents, Logs and Surds**

**Logs**

**Logs, Practice examples 1:**

*Simplify the following:*

1)  $\log_4 4$

2)  $\log_3 81$

3)  $\log_2 16 \times \log_2 8$

4)  $\frac{\log 81}{\log 27}$

5)  $\log_{\frac{1}{2}} 8$

*Solve for x:*

1)  $\log_4 x = 3$

2)  $\log_2 x = -2$

3)  $\log_3 27 + \log_4 x = 10$

4)  $\log 3 + \log 5 = \log x$

5)  $2^{\log x} = 16$

**Logs, exercise**

*Simplify:*

1)  $\log_3 81$

2)  $\log_5 \frac{1}{5}$

3)  $\log_2 16 - \log_2 8$

4)  $\frac{\log 81}{\log 27}$

5)  $\log \frac{3}{4} + 2 \log \frac{2}{5} - \log 12$

6)  $\frac{\log 36 - \log 9}{\log 3 - \log 6}$

7)  $\frac{\log^3 \sqrt[3]{81} - \log 27 + \log \frac{1}{3}}{\log 3^6 - \log 9}$

8)  $\frac{\log 6 - \log 2}{\log 9(2 \log 5 + \log 4)}$

*Solve for x:*

1)  $\log_3 x = 3$

2)  $\log_2 x = -2$

3)  $\log_{\frac{1}{4}} x = 2$

4)  $-\log_3 x = 3$

5)  $\log x^4 = 8$

6)  $\log_3(x - 3) - \log_3 5 = 1$

7)  $\log x + \log 4 - \log(2x - 3) = 2$

8)  $\log_5 x + \log_5 x^3 + \log_5 x^3 = 18$

9)  $\log x = \frac{\log 4}{\log 2}$

10)  $\log_x 36 = 2$

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**Exponents, Logs and Surds**  
**Cumulative**

**Cumulative practice examples 1:**

*Simplify:*

1)  $\frac{\sqrt{75}}{\sqrt{3}}$

2)  $\left(\frac{\sqrt[3]{x}}{\sqrt[2]{x^{-3}}}\right)^{\frac{1}{4}}$

3)  $\frac{18^x \cdot (2 \cdot 3^{-x})^3}{2^{x-2}}$

4)  $\frac{\log 4 - \log 25}{\log 2 - \log 5}$

5)  $\log \frac{3}{4} + 2 \log \frac{2}{5} - \log 12$

*Solve for x:*

1)  $2^{3x+3} + 4 \cdot 2^{3x} = 24$

2)  $\frac{(a^4)^{x-1} \cdot a}{\sqrt[x]{a}} = 1$

3)  $16 \cdot 2^x = \frac{4^{2x}}{2^{x-3}}$

4)  $\log_3 x + 2 \log_x 3 = 3$

5)  $\log_2 x + 1 = \log_2 (x + 1)$

**Cumulative exercise:**

*Simplify:*

1)  $\frac{8^n \times 4^{n+2}}{16^{4-n}}$

2)  $\frac{81^x + 9^{2x-2}}{18^x \cdot 2^{-x}}$

3)  $\frac{9 - \sqrt{54}}{6\sqrt{2}}$

4)  $\sqrt{3}(\sqrt{75} - \sqrt{3})$

5)  $\sqrt[n]{\frac{6^n + 2^n \cdot 25}{9^n + 25 \cdot 3^n}}$

6)  $3^{\frac{-1}{2}}(\sqrt{12} + 3\sqrt{3})$

7)  $\frac{\log_2 64}{\log 5 + \log 20}$

8)  $\sqrt[3]{27^2} + \log_2 4$

9)  $\log_9 27 + 2 \log_3 \sqrt{27}$

10)  $\log_3 2 \times \log_2 5 \times \log_5 9$

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Solve for x:

1)  $2^x + 2^{x-1} = 12$

2)  $x^{\frac{-4}{5}} = \sqrt[5]{16}$

3)  $9^x = (27^{3x-2})^{x-1}$

4)  $27^{\frac{2}{3}} + 16^{\frac{3}{4}} = 125^{\frac{2}{3}} + x$

5)  $2^{2x+1} - 3 \cdot 2^x + 1 = 0$

6)  $\frac{3 \cdot 2^{2x-1} - 2^{2x+1}}{4^x} = 2^{-3x} - 1$

7)  $\log_4 27 + \log_4 (x-1) = 1$

8)  $3 \log_5 x = 9$

9)  $\log_3 \sqrt{x} + \frac{1}{2} = \frac{\log 27}{\log x}$

10)  $6^x = 240$

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**Algebra 2**  
**Basic factorization**

**Algebra 2 practice examples 1**

*Factorize fully:*

1)  $x^2 + 4x - 32$

2)  $y^2 - 3y + 2$

3)  $a^2 + 6a$

4)  $6x + 9 + x^2$

5)  $y^2 + 18 - 9y$

6)  $2x^2 + 24x - 56$

7)  $a^2 - b^2$

8)  $-3x^2 + 24x + 60$

**Algebra 2 practice examples 2**

*Factorize fully:*

1)  $2x^2 - 5x - 12$

2)  $3x^2 - 9x + 6$

3)  $4x^2 - 33x - 70$

4)  $2x^2 - 9x - 5$

**Algebra 2 exercise:**

*Factorize fully:*

1)  $x^2 + 7x + 10$

2)  $x^2 - 5x - 24$

3)  $x^2 - 9x + 20$

4)  $x^2 + 13x - 30$

5)  $2x^2 + 5x + 3$

6)  $3x^2 + 11x - 20$

7)  $6x^2 + 7x - 20$

8)  $3x^2 - 10x + 8$

9)  $2x^2 - x - 10$

10)  $2x^3 - 2x^2 - 40x$

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Algebra 3  
Quadratic formula and completing the square

Algebra 3 practice examples 1

Solve for unknown variable by completing the square:

1)  $x^2 + 8x - 8 = 0$

2)  $2x^2 + 6x - 8 = 0$

3)  $3x^2 + 4x - 7 = 0$

4)  $2x^2 - 9x - 21 = 0$

5)  $x^2 - 2x + 20 = 0$

Algebra 3 practice examples 2

Solve for unknown variables by using the quadratic formula:

1)  $3x^2 - 3x - 10 = 0$

2)  $6x^2 - 2x - 12 = 0$

3)  $9x^2 + 28x - 100 = 0$

4)  $2x^2 + 4x - 100 = 0$

5)  $4x^2 + 2x + 8 = 0$

Algebra 3 exercise:

Solve, using any method:

1)  $2x^2 + 17x + 10$

2)  $4x^2 - 5x - 245$

3)  $9x^2 - 19x + 20$

4)  $12x^2 + 13x - 30$

5)  $2x^2 + 15x + 3$

6)  $31x^2 + 11x - 205$

7)  $6x^2 + 72x - 2$

8)  $-3x^2 - 10x + 84$

9)  $-12x^2 - 22x - 10$

10)  $2x^3 - 2x^2 - 40x$

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**Algebra 4**  
**Basic simultaneous**

**Algebra 4 practice examples 1**

Solve for unknown variables simultaneously:

- 1)  $x + y = 5$   
 $x^2 + 8y^2 - 15x = 6$
- 2)  $x - y - 7 = 10$   
 $3x^2 + 8y^2 - 15y = 7$
- 3)  $8x + 9y = 20$   
 $16x^2 + 18xy - 20 = 14x$
- 4)  $x + 2y - z = 4$   
 $x - 3z + 6y = 9$   
 $3x - 2y + 2z = 16$

**Algebra 4 exercise:**

Solve for unknown variables:

- 1)  $4x + 6y = 23$   
 $x^2 - 2y + 18 = 0$
- 2)  $3y^2 - 17x = 45$   
 $x^2 - 1 = y$
- 3)  $x^3 - 5 = 4y$   
 $y + 7 = -14$
- 4)  $x^2 + y^2 = 19$   
 $x^2 + 2y^2 = 23$
- 5)  $x - y = 2z$   
 $y + x = 12z$   
 $4 - z = x + y$

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Algebra 5  
Dealing with fractions

Algebra 5 practice examples 1

Solve for unknown variable:  
(Remember your LCD's)

1)  $\frac{3}{x} + 6x = 9$

2)  $\frac{2x+3}{x+1} - \frac{6}{x} = 9$

3)  $\frac{x+6}{x-4} + \frac{5}{x^2+2x-24} = \frac{5}{(x+6)(x-4)} + \frac{3x-12}{x+6}$

4)  $\frac{x^2-4x}{2} + \frac{3}{x(x-4)} = 3\frac{1}{2}$

5)  $\frac{2}{4x^2-1} + \frac{1}{1-2x} = \frac{x}{2x+1}$

Algebra 5 exercise:

Solve for unknown variable:

1)  $\frac{8}{x} + 5x = 15$

2)  $\frac{2x+9}{3x-9} - \frac{4}{2x} = 6$

3)  $\frac{x-3}{x+5} + \frac{4}{x^2+x-20} = \frac{7}{(x+5)(x-4)} + \frac{x-2}{x-4}$

4)  $\frac{x^2-6x}{3} + \frac{2}{x(x-6)} = 4$

5)  $\frac{2}{9x^2-4} + \frac{1}{2-3x} = \frac{4x}{3x+2}$

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**Algebra 6**  
**Inequalities basic**

**Algebra 6 practice examples 1**

*Find the value(s) of the unknown variables:*

1)  $\frac{9x-4}{3x+1} \geq 2$

2)  $3x+7 \leq \frac{-6x-4}{5}$

3)  $\frac{x+4}{x} \geq \frac{3x}{x+2}$

4)  $\frac{9x+3}{12(x+2)} + \frac{3x}{x+1} \geq \frac{6x}{x+2}$

5)  $\sqrt{2x+12} \geq x-6$

**Algebra 6 exercise:**

*Find the value(s) of the unknown variables:*

1)  $\frac{9x^2-1}{3x+1} \geq 8$

2)  $5x-9 \leq \frac{x-24}{6}$

3)  $\frac{2x+8}{4x} \geq \frac{2x}{7x+7}$

4)  $\frac{9x-6}{12(x+3)} + \frac{5x}{5x+25} \geq \frac{7x}{x^2+8x+15}$

5)  $\sqrt{4x-21} \geq x+7$

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**Functions 1**  
**Basic functions**

**Functions 1 practice examples 1**

*Using the tables provided, sketch the following functions:*

1)  $f(x) = 4x + 3$

$x$	<b>-3</b>	<b>-2</b>	<b>-1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
$f(x)$							

2)  $g(x) = -3x + 1$

$x$	<b>0</b>	
$h(x)$		<b>0</b>

3)  $h(x) = 3x + 5$

$x$	<b>-2</b>	<b>-1</b>	<b>0</b>	<b>1</b>	<b>2</b>
$g(x)$					

4)  $j(x) = \frac{3}{x}$

$x$	<b>-6</b>	<b>-3</b>	<b>-1</b>	<b>0</b>	<b>1</b>	<b>3</b>	<b>6</b>
$j(x)$							

5)  $k(x) = 2^x$

$x$	<b>-3</b>	<b>-2</b>	<b>-1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
$k(x)$							

*Sketch the following functions using any method*

6)  $A(x) = 2x - 1$

7)  $B(x) = -2x + 3$

8)  $C(x) = 4^x$

9)  $D(x) = 4^{-x}$

10)  $E(x) = \frac{1}{2x}$

**Functions 1 practice examples 2**

*Think of possible scenarios to match the following functions:*

*(Note that there is no one right answer, what I do on-screen is merely an illustration)*

1)  $g(x) = \frac{2}{13}x + 9$

2)  $h(x) = 6^x - 240$

3)  $l(x) = \frac{1240}{x}$

**Functions 1 exercise:**

*Sketch the following:*

1)  $A(x) = 3x + 5$

2)  $B(x) = \frac{2}{x}$

3)  $j(x) = 2^x$

**Functions 2**  
**Straight line**

**Functions 2 practice examples 1**

Sketch the following functions:

- |    |                   |    |                   |
|----|-------------------|----|-------------------|
| 1) | $2y = 9 - 8x$     | 2) | $3x + 5y = -6$    |
| 3) | $y = 3x + 4y - 5$ | 4) | $3x + y = 3x - 1$ |
| 5) | $2x = 4$          | 6) | $3x = 9y$         |

**Functions 2 exercise:**

Sketch the following:

- |    |                        |    |                         |
|----|------------------------|----|-------------------------|
| 1) | $y = 5x + 1$           | 2) | $y = 4x - 8$            |
| 3) | $y = -2x + 4$          | 4) | $y = -3x - 1$           |
| 5) | $y = \frac{1}{4}x + 8$ | 6) | $y = \frac{-7}{8}x - 5$ |
| 7) | $2y = 4x - 16$         | 8) | $3y = 5x - 1$           |
- 

**Functions 3**  
**The parabola**

**Functions 3 practice examples 1**

Sketch the following functions using steps:

- |    |                                   |    |                                    |
|----|-----------------------------------|----|------------------------------------|
| 1) | $g(x) = (x - 3)^2 + 4$            | 2) | $f(x) = -2(x - 3)^2 + 8$           |
| 3) | $j(x) = \frac{1}{4}(x + 1)^2 - 8$ | 4) | $k(x) = \frac{-1}{3}(x + 2)^2 + 2$ |
| 5) | $h(x) = -2x^2 + 8x - 16$          |    |                                    |

**Functions 3 practice examples 2**

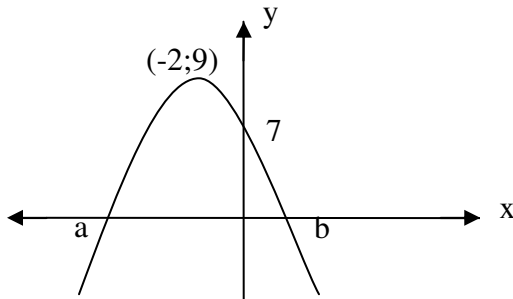
Sketch the following functions using steps:

- |    |                         |    |                          |
|----|-------------------------|----|--------------------------|
| 1) | $f(x) = 2x^2 - 8x - 24$ | 2) | $g(x) = -4x^2 - 4x + 24$ |
| 3) | $h(x) = (x - 5)(x + 6)$ | 4) | $l(x) = 2(x - 1)^2 + 5$  |
| 5) | $k(x) = -4x^2$          |    |                          |

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Functions 3 practice examples 3

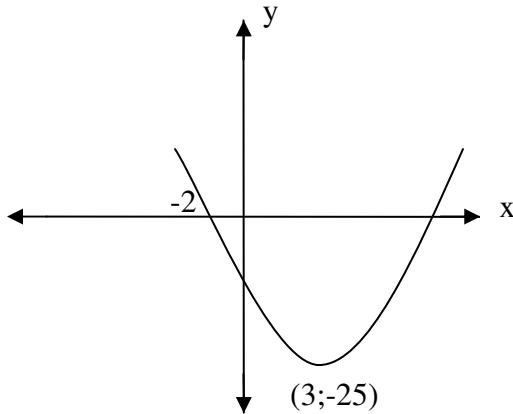
1)



$$b(x) = a(x - p)^2 + q$$

- 1.1) determine a, p and q
- 1.2) find distance AB

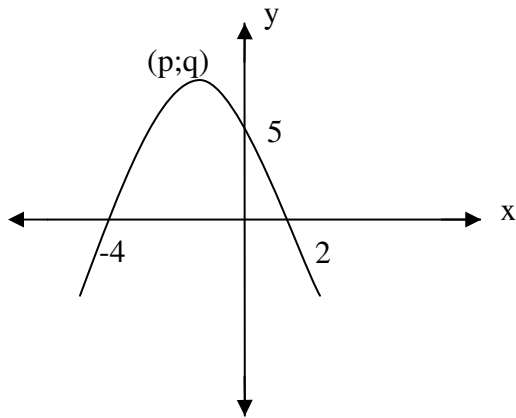
2)



$$c(x) = ax^2 + bx + c$$

- 2.1) determine a, b and c
- 2.2) the other x intercept
- 2.3) the y intercept

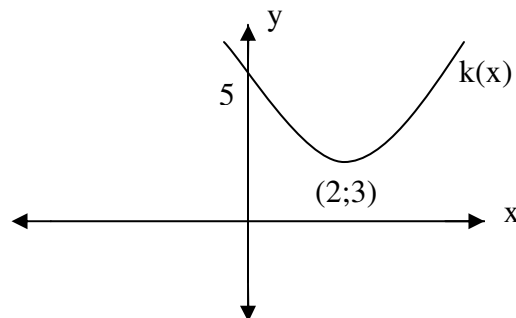
3)



$$h(x) = a(x - p)^2 + q$$

- 3.1) find a, p and q

4)



- 4.1) determine the equation  $k(x)$
- 4.2) can  $k(x) = 0$ ?

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**Functions 3 exercise:***Sketch the following:*

1)  $f(x) = 3x^2 - 12x - 36$

2)  $g(x) = -2x^2 + 16x - 24$

3)  $h(x) = \frac{1}{5}(x+4)(x-8)$

4)  $j(x) = (x+5)^2 + 4$

5)  $k(x) = 3x^2 + 12x$

6)  $m(x) = -5x^2$

**Functions 4**  
**Hyperbola****Functions 4 practice examples 1:***Sketch the following:*

1)  $c(x) = \frac{3}{x}$

2)  $d(x) = \frac{-2}{x+5} - 4$

3)  $e(x) = \frac{3}{x-1}$

4)  $f(x) = \frac{-2}{x} + 2$

**Functions 4 exercise:***Sketch the following:*

1)  $f(x) = \frac{-2}{x}$

2)  $g(x) = \frac{3}{x+2} - 7$

3)  $h(x) = \frac{1}{x-1}$

4)  $j(x) = \frac{-3}{-x} + 2$

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**Functions 5**  
**Exponential**

**Functions 5 practice examples 1**

*Sketch the following:*

1)  $f(x) = 2^x$   
3)  $h(x) = 2^{x+2}$

2)  $g(x) = 3^{-x}$   
4)  $j(x) = 4^x - 3$

**Functions 5 practice examples 2**

*Sketch the following:*

1)  $y = 3 \log_2 x$   
3)  $5x = 2^y$

2)  $y = \log_{10} x$

**Functions 5 exercise**

*Sketch the following, indicating all intercepts with the axis:*

1)  $f(x) = 3^{x+4} - 3$   
3)  $h(x) = -2 \log_3 x$

2)  $g(x) = -(4^{x-3}) + 2$   
4)  $j(x) = 2 \log_4 x$

**Function 6**  
**Trigonometric graphs**

**Functions 6 practice examples 1:**

*Sketch the following using the table/s provided:*

1)  $y = \cos x$

x	-360°	-270°	-180°	-90°	0°	90°	180°	270°	360°
y									

2)  $y = \tan x$

x	-360°	-270°	-180°	-90°	0°	90°	180°	270°	360°
y									

x	-360°	<b>-315°</b>	-270°	<b>-225°</b>	-180°	<b>-135°</b>	-90°	<b>-45°</b>	0°
y									
x	<b>45°</b>	90°	<b>135°</b>	180°	<b>225°</b>	270°	<b>315°</b>	360°	
y									

**Functions 6 exercise:**

*Sketch the graph of  $y = \sin x$*

**Calculus 1**  
**Basic differentiation**

**Calculus 2 practice examples 1**

*Differentiate using first principals:*

- 1)  $f(x) = 3x + 4$   
2)  $g(x) = 4x^2 + 6x + 8$

*Differentiate using any method:*

- 3)  $y = 9x^2 + 18x - 16$   
4)  $f(x) = \frac{4}{\sqrt{x}} + 9x^{10} - \frac{15}{4}\sqrt[4]{x}$   
6)  $g(x) = 12x^2 - \frac{6}{3x^4} + \frac{18x}{x^2}$

**Calculus 1 exercise:**

*Differentiate using first principals:*

- 1)  $f(x) = 2x - 5$   
2)  $g(x) = 2x^2 - 16x + 7$

*Differentiate using any method:*

- 3)  $y = 7x^2 + 7x - 12$   
4)  $f(x) = \frac{4 - x^3}{\sqrt{x}} - 15x$   
7)  $g(x) = -2x^{14} + \frac{3}{45x^{32}} + \frac{18\sqrt{x} + x}{x^2}$

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**Calculus 2****Long division, remainder & factor theorem****Calculus 2 practice examples 1**

- 1) If  $(x - 2)$  is a factor of  $g(x) = x^3 - x^2 - 22x + 40$ , solve for  $g(x) = 0$
- 2) Use the remainder theorem to find the remainder when  $f(x) = x^3 - 6x^2 + 8x - 5$  is divided by  $h(x) = 3x + 4$
- 3) If  $f(x) = -3x^3 + 4x^2 - mx + 6$  is divisible by:  $g(x) = 2x + 8$ , what is the value of  $m$ ?
- 4) Solve for  $x$ :
  - 4.1)  $g(x) = x^3 + 2x^2 - 20x + 24 = 0$
  - 4.2)  $h(x) = x^4 - 7x^3 - x^2 + 7x = 0$
  - 4.3)  $f(x) = x^3 + 4x^2 - 9 = 0$
  - 4.4)  $k(x) = x^3 + 3x^2 + 3x + 1 = 0$

**Calculus 2 exercise:**Solve for  $x$ :

- 1)  $2x^3 + 5x^2 - 19x - 42 = 0$
- 2)  $x^3 + 9x^2 + 27x + 27 = 0$
- 3)  $x^4 - 4x^3 - 7x^2 + 22x + 10 = 0$
- 4) what is the remainder when  $f(x) = 8x^3 - 5$  is divided by  $g(x) = 2x + 3$

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Calculus 3  
3<sup>rd</sup> degree (cubic) graphs

Calculus 3 practice examples 1

Sketch the following graphs:

1)  $g(x) = x^3 - 7x^2 - x + 7$

2)  $h(x) = -2x^3 - 6x^2 + 56x + 120$

Calculus 3 exercise:

Sketch the following graphs:

1)  $f(x) = 2x^3 + 5x^2 - 19x - 42$

2)  $g(x) = -x^3 - 9x^2 - 27x - 27$ 

---

**Financial Mathematics 1**  
**Basic financial mathematics**

*Financial Math 1 Exercise 1*

- 1) What will the value of a R100 000 investment be in 5 years time if interest is calculated at 16%p.a., simple interest?
  - 2) How much would you have to invest today to receive a payout of R1 000 000 in ten years time if our investment would yield 18% p.a., simple interest?
  - 3) If interest is compounded annually, after 20 years, what would an investment of R200 000 yield if the interest rate is 17% p.a.?
  - 4) Assuming inflation to be a constant 7% p.a. what would our same investment be worth (of 3 above)?
  - 5) Interest is measured at 16% p.a., compounded monthly. How much should be invested to yield a payout of R1 200 000 in 15years from now?
  - 6) If we intend to settle an outstanding balance of R50 000 that is owed in 28months time, how much would we have to pay now to settle the amount? Assuming interest at 15% p.a., compounded monthly.
- 

**Financial Maths 2**  
**Timelines**

*Financial Maths 2, exercise:*

- 1) If we invest R200 today, R800 in a years time and R2000 in two years time, how much will we have at the end of 25years if our investment earns an interest of 18% p.a. compounded annually?
- 2) A mother wishes to give each of her three kids a gift of R200 000 on each of their 21<sup>st</sup> birthdays. Her eldest kid is 8 years old, the second is 4 and the youngest is two. How much should she invest today if the investment option available to her yields an interest rate of 16% p.a. compounded quarterly?
- 3) We start an investment with an amount of R200 000. In a years time we deposit a further R200 000 and another R100 000 in two years time. If after 3 years we draw R150 000, what will our investment be worth at the end of 5years if interest rate is 14% p.a. compounded twice yearly?
- 4) Of 3 above, what would our investment be worth had we not withdrawn the R150 000?

**Financial Maths 3**  
**Annuities and loans**

*Financial Maths 3, Exercise*

- 1) If we deposit R300 monthly into an annuity paying out 17% p.a., compounded monthly, how much would we have at the end of 20 years?
  - 2) Annual payments are made by a parent into a fund yielding 16% p.a., compounded yearly. If the first payment is on the child's first birthday, how much would the fund be worth on the child's 21<sup>st</sup> birthday?
  - 3) Using the same scenario as above, imagine the child takes over payments at the age of 21 and continues until their 65<sup>th</sup> birthday, how much would the fund then be worth on their 65<sup>th</sup> birthday?
  - 4) You take out a loan to buy a car costing R100 000. How much will your monthly repayments be over five years if interest is calculated at 13% p.a. compounded monthly?
  - 5) Using the same scenario as above, you are able to put down a 25% deposit on the car. What will your new monthly repayments be?
  - 6) If you can afford a monthly repayment of R2000, with interest at 12% p.a., compounded monthly, what size loan may you be granted over a 20 year period?
-

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**Number patterns 1**  
**Arithmetic sequences**

*Number patterns 1, practice examples 1:*

- 1) Determine the general solution of the following  
4; 6; 8.....
- 2) Determine the general solution of the following:  
9; 6; 3; 0.....
- 3) Of the series; 7; 9.5; 12; ... what is the 23<sup>rd</sup> term?
- 4) Of the series; 10; 9.7; 9.4; ... what is the 7<sup>th</sup> term?
- 5) Of the series; 5; 10; 15; 20; ... which term is 100?
- 6) Of the series; 66; 58; 50; ... which term is the first term that is less than zero?
- 7) If the first, second and third terms add up to 210 and the fourth term is 116, what is the first term, the common difference and the general solution?

*Number patterns 1, practice examples 2:*

- 1) What is the sum of the first 20 terms of the series:  $1 + 2 + 3 + 4 + \dots$ ?
- 2) What is the sum of the first 800 terms of the series:  $5 + -1 + -7 + \dots$ ?
- 3) How many terms of the series 2; 2; 2; ... must be added to reach a total of 500?
- 4) How many terms must be added in the series  $100 + 92 + 84 + \dots$  to reach a total of -600?
- 5) 52 is the sum of ten terms of the series  $x + y + z$ . Determine the values of  $x$ ,  $y$  and  $z$  if the 7<sup>th</sup> term is 18.

*Number patterns 1, practice examples 2:*

*Determine the solution of the following:*

- |                              |                                     |
|------------------------------|-------------------------------------|
| 1) $\sum_{i=4}^{n=3} 3i - 1$ | 2) $\sum_{i=2}^{n=8} 2i - 3(i - 1)$ |
| 3) $\sum_{i=0}^{n=0} 2i - 1$ | 4) $\sum_{i=1}^{n=10} 6i$           |

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Number patterns 1, exercise 1:

- 1) Determine the general solution of the series: 45; 47; 49; ... and the sum of 15 terms
  - 2) Determine the general solution of the following series: -5; -15; -20; ... and the sum of 104 terms
  - 3) What is the 30<sup>th</sup> term of the series starting at -7, with a common difference of 8? And determine the sum of 32 terms
  - 4) If the fourth term of an arithmetic series is 12, the sixteenth term is 108, determine the 25<sup>th</sup> term and the sum of 25 terms
  - 5) Solve:  $\sum_{i=3}^{n=18} 3i - 2(i - 1)$
- 

**Number patterns 2**  
**Geometric sequences**

Number patterns 2, practice examples 1:

- 1) In the geometric sequence: 2; 4; 8; ... determine the general solution as well as the 20<sup>th</sup> term.
- 2) Of the series: 81; 27; 9; ... determine the general solution as well as the 9<sup>th</sup> term.
- 3) Consider the series:  $x$ ;  $x^2$ ;  $x^3$ :
  - 3.1) What is the ratio?
  - 3.2) For which r values will the series diverge?
  - 3.3) For which r values will the series converge?
- 4) Of the series:  $\frac{2}{8}$ ;  $\frac{1}{8}$ ;  $\frac{1}{16}$ , which term is  $\frac{1}{1024}$ ?
- 5) How many terms in the series: 2; 4; 8; ... lie between 8 and 2048?

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Number patterns 2, practice examples 2:

- 1) Determine the solution of:  $\sum_{i=1}^{n=8} 3^i$
- 2) Determine the solution of:  $\sum_{i=2}^{n=15} \left(\frac{2}{5}\right)^i$
- 3) Of the series  $3 + \frac{3}{5} + \frac{3}{25} + \dots$  how many terms need to be added to reach a total of  $3\frac{93}{125}$  ?
- 4) Consider the series:  $\frac{3}{4} + \frac{6}{12} + \frac{12}{36}$ 
  - 4.1) What is the sum of 20 terms?
  - 4.2) What is the sum of 200 terms?
  - 4.3) What is the sum of 2000 terms?
  - 4.4) Is the series convergent or divergent?

Number patterns 2, practice examples 3:

- 1) Find the sum to infinity of the series:  $\frac{3}{4} + \frac{6}{12} + \frac{12}{36}$
- 2) Determine the solution of:  $\sum_{i=4}^{\infty} \left(\frac{3}{5}\right)^i$
- 3) Determine the solution of:  $\sum_{i=3}^{\infty} 2^i$

Number patterns 2, practice exercise:

- 1) Determine the general solution of the series: 14; 28; 56; ... and the sum of 10 terms
- 2) Determine the general solution of the following series: -8; 16; -32; ... and the sum of 100 terms
- 3) What is the 20<sup>th</sup> term of the series starting at 4, with a common ratio of 8? And determine the sum of 25 terms
- 4) If the fourth term of a geometric series is 12, the sixteenth term is 108, determine the 25<sup>th</sup> term and the sum of 25 terms

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5) Solve:  $\sum_{i=2}^{n=15} \left(\frac{12}{11}\right)^i$

6) Solve:  $\sum_{i=2}^{n=\infty} \left(\frac{2}{5}\right)^i$

**Number patterns 3**

**2<sup>nd</sup> Difference:**

*Number patterns 3, practice examples 1:*

Consider the following sequence: 6; 11; 18; ...

- 1) determine the next two terms
- 2) find the general solution

*Number patterns 3, exercise:*

1) Consider the following sequence: 3; 4; 9; 18; ...

- a) determine the next two terms
- b) find the general solution

2) Consider the following sequence: 7; 5; 1; -5; ...

- a) determine the next two terms
  - b) find the general solution
-

**Linear Programming**

*Linear programming, exercise:*

- 1) A factory manufactures two types of beds, namely foams and springs (x and y).

Suppose the factory manufactures according to the following constraints:

$$\begin{array}{rcl} x + y & \leq & 50 \\ x & \leq & 35 \\ 2x + y & \geq & 30 \\ y & \geq & 10 \end{array}$$

The profit per foam (x) is R150 and R120 per spring (y),

- a) Sketch the inequalities and determine the feasible region
  - b) Determine the maximum profit function
  - c) Find the point that maximizes profit
  - d) Calculate the maximum profit
- 2) According to a recommended diet plan, at least 10mg of vitamin A and 16mg of vitamin B must be taken daily. A tablet (x) contains 2mg of vitamin A and 1mg of vitamin B. A capsule (y) contains 1mg of vitamin A and 4mg of vitamin B.

- a) Determine the inequalities using the following table:

vitamin	A	B
Tablet (x)	2	1
Capsule (y)	1	4
Min req.	10	16

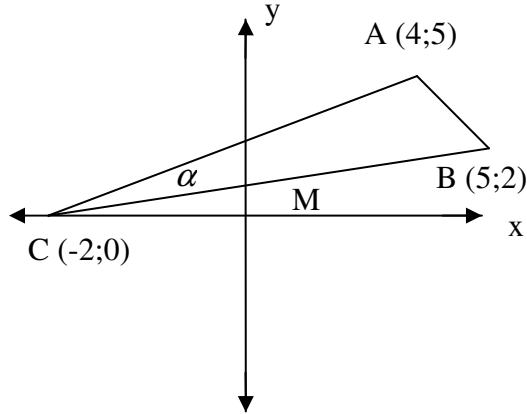
- b) Sketch the inequalities and determine the feasible region
  - c) What is the minimum amount of tablets one needs to take?
- 3) A company makes two types of chairs, X and Y. The company only has 200m<sup>2</sup> of storage space available. X requires 1.5m<sup>2</sup> of storage space and Y requires 1m<sup>2</sup>. The company can only afford to invest R20 000 in equipment, costs are R120 per X and R180 per Y. Not more than 50 of Y may be manufactured per day due to a glue drying process.  
Profit per X is R150 and R130 per Y.
- a) Determine and sketch the inequalities
  - b) Find the feasible region
  - c) Determine the objective profit function
  - d) Determine the point that will yield the maximum profit
  - e) Determine the maximum profit

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**Analytical Geometry 1**  
**Basics**

*Analytical Geometry 1, practice examples 1:*

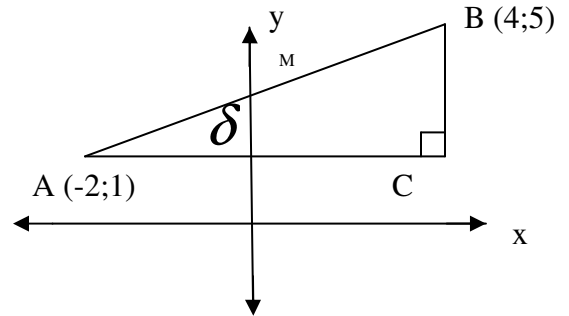
1)



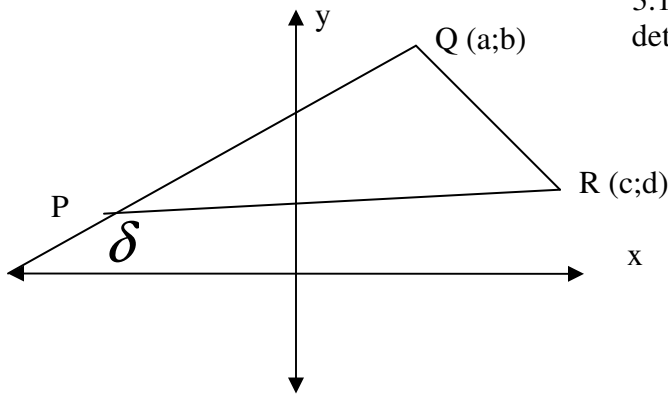
- 1.1) Determine the lengths of BC, AB and AC
- 1.2) Is  $\triangle ABC$  a right angle triangle with BC the hypotenuse?
- 1.3) What are the co-ordinates of M, the midpoint of BC?
- 1.4) what is the value of angle  $\alpha$  ?

2)

- 2.1) What is the value of  $\delta$  ?
- 2.2) determine the co-ordinates of M, the midpoint of AB
- 2.3) Determine the distance CM if  $AC = BC$



3)



- 3.1) If  $PQ = 8$  units, and  $\delta$  is  $30^\circ$ , determine the values of a and b.
- 3.2) what would c and d 's values have to be for  $\triangle PQR$  to be an equilateral triangle?

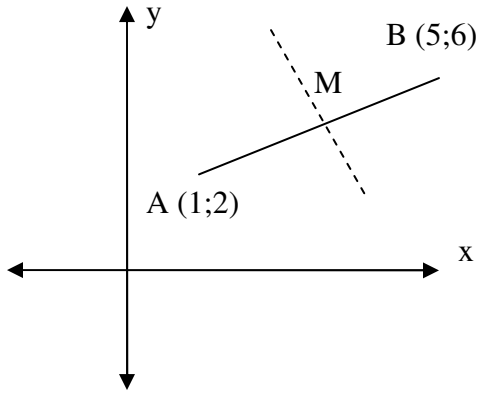
*Analytical Geometry 1, exercise:*

- 1) Sketch the following points on a cartesian plane: A(2; 5); B(3; -6); C(-5; 2); D(-4;-2)
- 2) Using the above sketched points, determine:
  - a. The distances AB and CD
  - b. The gradients of AC and AB and their respective angles of inclination
  - c. Calculate the co-ordinates of E such that it is the midpoint of AD
  - d. What is the gradient of EF if it is perpendicular to AD and goes through E

**Analytical Geometry 2**  
**Equations and gradients**

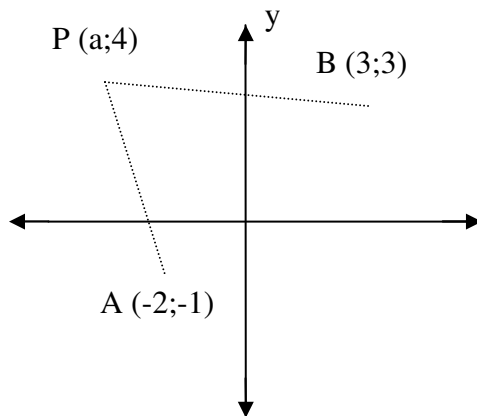
*Analytical Geometry 2, practice examples 1;*

1)



- 1.1) determine the length of AB
- 1.2) what are the co-ordinates of M, the midpoint of AB
- 1.3) Equation of AB
- 1.4) Equation of perpendicular bisector through M

2)



- 2.1) Point P is equidistant from A and B. What is the value of a?

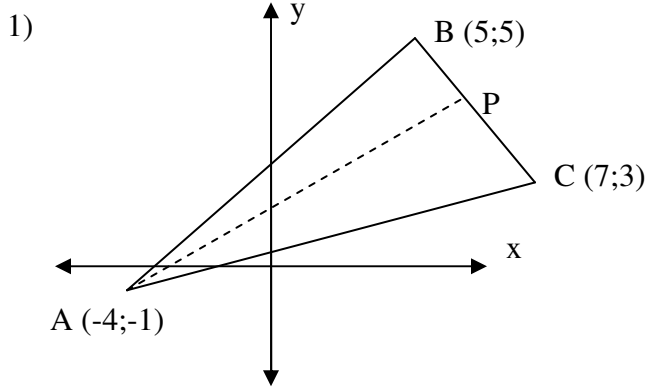
*Analytical geometry 2, exercise:*

- 1) Sketch the following points on a cartesian plane: A(2; 5); B(3; -6); C(-5; 2); D(-4;-2)
- 2) Using the above sketched points, determine:
  - a. The equations of AB and CD
  - b. The gradients of AC and AB and their respective angles of inclination
  - c. Calculate the co-ordinates of E such that it is the midpoint of AD
  - d. What is the equation of EF if it is the perpendicular bisector of AD
- 3) Determine the co-ordinates of F such that it lies on the line BC as well

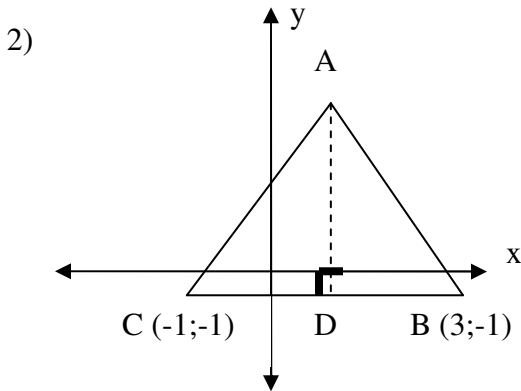
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**Analytical geometry 3**  
**Triangles: medians, midpoints and heights.**

*Analytical Geometry 3, practice examples 1:*



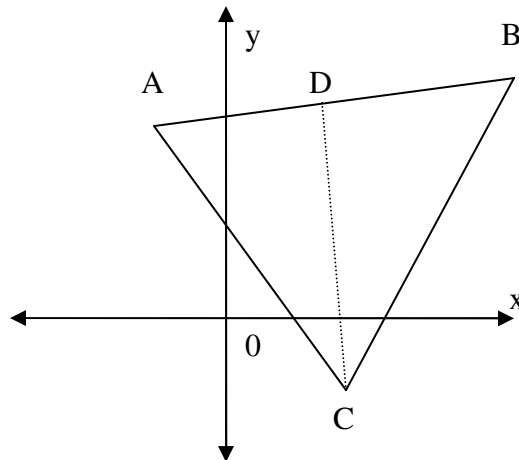
- 1.1) Determine equation of AP if AP is a Median
- 1.2) Length of AP
- 1.3) Is AP also a height of the triangle?



- 2.1) If  $AB = AC$ , determine the equation of AD. ( $AD \perp CB$ )
- 2.2) Is AD a median as well as a height?

*Analytical geometry 3, exercise:*

- 1) If the equation of ac is given by:  
 $2y = -3x + 6$ , determine the co-ordinates of C and D such that CD is the median and height of Triangle ABC.

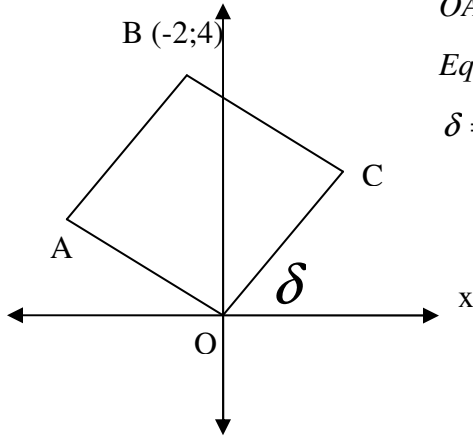


**Analytical Geometry 4**  
**Basic cumulative**

*Analytical Geometry 4, Practice examples 1:*

- 1) Given the points: P ( 6 ; 7 ), Q ( 1 ; -7 ) and R ( 3 ; a )  
 1.1) Determine the length of PQ  
 1.2) Determine the value of a if  $PQ = 3 PR$

2)



*OABC is a parallelogram ( $AB \parallel OC$  and  $BC \parallel OA$ )*

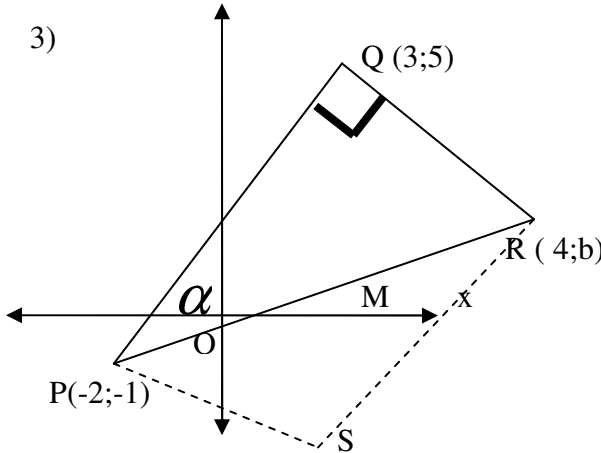
*Equation of OA given by:  $y = \frac{-1}{2}x$*

$\delta = 45^\circ$

Determine:

- 2.1) the equation of CB  
 2.2) the equation of AB  
 2.3) the co-ordinates of A and C  
 2.4) distance of AC  
 2.5) size of angle AOC

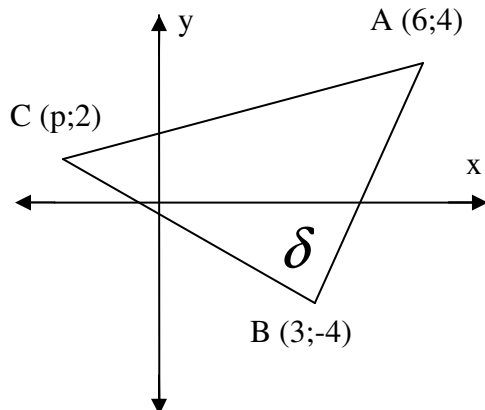
3)



Determine:

- 3.1) value of b, the y-co-ordinate of R  
 3.2) co-ordinates of M, the midpoint of PR  
 3.3) size of  $\alpha$  (the angle RPQ)  
 3.4) co-ordinates of S such that PQRS is a rectangle.

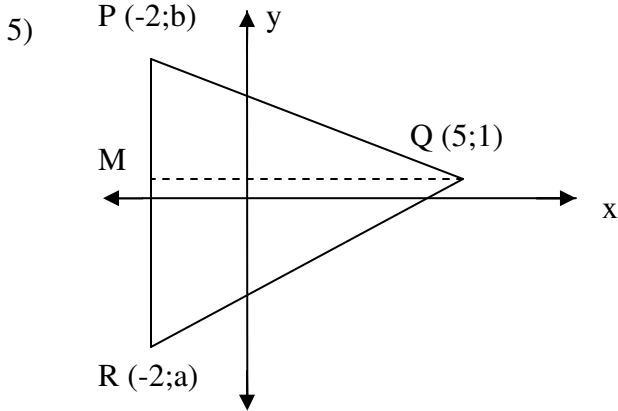
4)



4.1) calculate the value of p such that C and B are equidistant from A

4.2) Calculate the size of  $\delta$

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PQR is an equilateral triangle.  
 PQ cuts the y axis at 3  
 QR cuts the y axis at -2

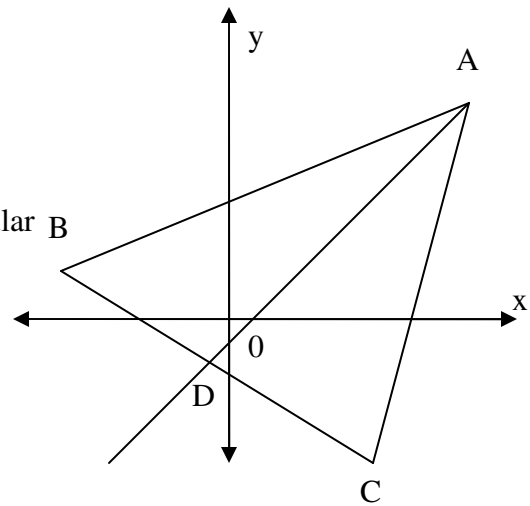
- 5.1) find values of a and b
- 5.2) is QM a height of triangle?
- 5.3) what are co-ordinates of M, the midpoint of PR?

Analytical geometry 4, exercise:

1) The co-ordinates of the triangle are as follows:

A (5;5), B(-7;1), C (1;-7)

- a. show that ABC is an isosceles triangle
- b. determine equation of AD, the perpendicular bisector of BC
- c. determine the equation of the line through A and perpendicular to AD.

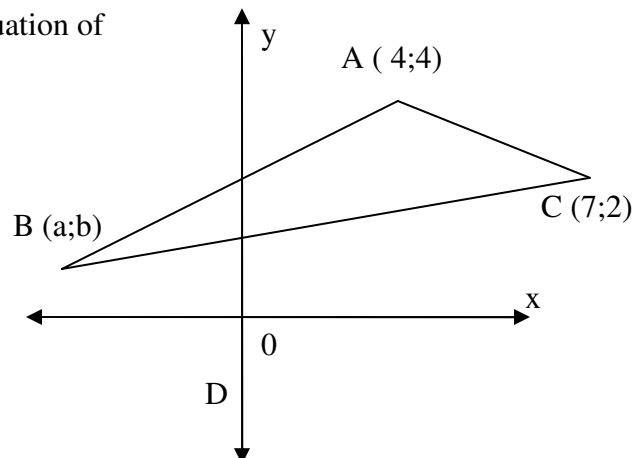


2) A (-1;4), B(-2;-2), C (3;2) are the vertices of a triangle.

- a. Determine the equation of the median BE
- b. Calculate  $\hat{A}BC$
- c. Determine co-ordinates of D if ABCD is a parallelogram
- d. Prove that D is collinear to BE (i.e.: lies on the same line)

3) Find the values of a and b if the equation of AB:  $y = x + 2$  and CB:  $2y = x + 1$

What is size of angle ABC?



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**Trigonometry 1**  
**Basics**

Trigonometry 1, practice examples 1:

*Solve for the following to two decimal places:*

- |                    |                    |
|--------------------|--------------------|
| 1) $\sin 40^\circ$ | 2) $\cos 80^\circ$ |
| 3) $\tan 20^\circ$ | 4) $\sin 50^\circ$ |
| 5) $\cos 40^\circ$ |                    |

*Solve for x to two decimal places:*

- |                   |                     |
|-------------------|---------------------|
| 1) $\sin x = 0.5$ | 2) $\cos x = 0.5$   |
| 3) $\tan x = 5$   | 4) $\cos x = 0.866$ |
| 5) $\tan x = 1$   |                     |

Trigonometry 1, practice examples 2:

*Sketch the following on the same set of axis:*

- |                |                |
|----------------|----------------|
| 1) $20^\circ$  | 2) $40^\circ$  |
| 3) $160^\circ$ | 4) $230^\circ$ |
| 5) $315^\circ$ |                |

Trigonometry 1, practice examples 3:

*Sketch right-angled triangles for the following:*

- |                                |                                 |
|--------------------------------|---------------------------------|
| 1) $\sin \delta = \frac{4}{5}$ | 2) $\cos \delta = \frac{-3}{6}$ |
| 3) $\tan \alpha = \frac{5}{3}$ | 4) $\sin \beta = \frac{4}{5}$   |
| 5) $\tan x = \frac{-8}{-5}$    |                                 |

Trigonometry 1, exercise:

*Solve to two decimal places:*

- |                     |                      |                     |                      |
|---------------------|----------------------|---------------------|----------------------|
| 1) $\sin 30^\circ$  | 2) $\tan 47^\circ$   | 3) $\cos 87^\circ$  | 4) $\sin 458^\circ$  |
| 5) $\cos 875^\circ$ | 6) $\tan 4455^\circ$ | 7) $\sin -97^\circ$ | 8) $\cos -720^\circ$ |

*Solve for x to one decimal place:*

- |                        |                         |                 |
|------------------------|-------------------------|-----------------|
| 1) $\sin x = 57^\circ$ | 2) $\cos x = 788^\circ$ | 3) $\tan x = 1$ |
|------------------------|-------------------------|-----------------|

*Sketch the following on a set of axis:*

- |                                 |                                  |                       |
|---------------------------------|----------------------------------|-----------------------|
| 1) $\sin \delta = \frac{-3}{6}$ | 2) $\cos \delta = \frac{8}{-12}$ | 3) $\tan \delta = -4$ |
|---------------------------------|----------------------------------|-----------------------|

**Trigonometry 2**  
**Reduction**

Trigonometry 2, Practice examples 1:

*Reduce the following using the x-reductions to acute angles:*

- |                     |                     |
|---------------------|---------------------|
| 1) $\sin 150^\circ$ | 2) $\cos 210^\circ$ |
| 3) $\tan 330^\circ$ | 4) $\cos 110^\circ$ |
| 5) $\sin 190^\circ$ | 6) $\sin 290^\circ$ |
| 7) $\tan 165^\circ$ | 8) $\tan 230^\circ$ |
| 9) $\cos 346^\circ$ |                     |

Trigonometry 2, Practice examples 2:

*Reduce the following using the y-reductions to acute angles:*

- |                     |                     |
|---------------------|---------------------|
| 1) $\sin 150^\circ$ | 2) $\cos 210^\circ$ |
| 3) $\tan 330^\circ$ | 4) $\cos 110^\circ$ |
| 5) $\sin 190^\circ$ | 6) $\sin 290^\circ$ |
| 7) $\tan 165^\circ$ | 8) $\tan 230^\circ$ |
| 9) $\cos 346^\circ$ |                     |

Trigonometry 2, Practice examples 3:

*Reduce the following to acute angles:*

- |                      |                       |
|----------------------|-----------------------|
| 1) $\sin 362^\circ$  | 2) $\tan 380^\circ$   |
| 3) $\cos 425^\circ$  | 4) $\tan 542^\circ$   |
| 5) $\cos 1024^\circ$ | 6) $\sin -380^\circ$  |
| 7) $\tan -261^\circ$ | 8) $\cos -59^\circ$   |
| 9) $\sin -800^\circ$ | 10) $\cos -500^\circ$ |

Trigonometry 2, exercise:

*Reduce the following to acute angles:*

- |                      |                      |
|----------------------|----------------------|
| 1) $\sin 32^\circ$   | 2) $\tan 745^\circ$  |
| 3) $\cos 42^\circ$   | 4) $\tan 485^\circ$  |
| 5) $\cos 365^\circ$  | 6) $\sin -45^\circ$  |
| 7) $\tan -287^\circ$ | 8) $\cos -789^\circ$ |
| 9) $\sin -751^\circ$ | 10) $\cos -3^\circ$  |

**Trigonometry 3**  
**Special angles**

Trigonometry 3, practice examples 1:

Solve for the following without the use of the calculator:

- 1)  $\sin 90^\circ$
- 2)  $\tan 45^\circ$
- 3)  $\cos 0^\circ$
- 4)  $\tan 60^\circ$
- 5)  $\cos 30^\circ$

Trigonometry 3, practice examples 2:

Solve for the unknown angles without the use of a calculator:

- 1)  $\cos \delta = \frac{\sqrt{3}}{2}$
- 2)  $\tan \delta = \frac{1}{\sqrt{3}}$
- 3)  $\cos \delta = 1$
- 4)  $\sin \delta = \frac{\sqrt{2}}{2}$
- 5)  $\sin \delta = 0$

Trigonometry 3, practice examples 3:

Solve for the following without the use of a calculator:

- 1)  $\sin 225^\circ$
- 2)  $\cos 300^\circ$
- 3)  $\tan 480^\circ$
- 4)  $\tan -120^\circ$
- 5)  $\sin -225^\circ$
- 6)  $\cos -750^\circ$

Trigonometry 3, exercise:

Solve for the following without the use of a calculator:

- 1)  $\sin 45^\circ$
- 2)  $\cos 90^\circ$
- 3)  $\tan 60^\circ$
- 4)  $\tan 240^\circ$
- 5)  $\sin 240^\circ$
- 6)  $\cos 150^\circ$
- 7)  $\cos 420^\circ$
- 8)  $\sin 510^\circ$
- 9)  $\tan -135^\circ$
- 10)  $\sin -510^\circ$

**Trigonometry 4**  
**Identities**

*Trigonometry 4, practice examples 1:*

*Prove the following identities:*

$$1) \quad \frac{\tan x(1 - \sin^2 x) - \cos^2 x}{\sin^2 x - \cos^2 x} = \frac{\cos x}{\sin x + \cos x}$$

$$2) \quad \frac{\sin x - \sin x \cdot \cos x}{\cos x - (1 - \sin^2 x)} = \tan x$$

$$3) \quad \left( \frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)^2 = \frac{1 - \cos x}{1 + \cos x}$$

$$4) \quad \frac{\cos 2x}{\cos^2 x} = (1 + \tan x)(1 - \tan x)$$

$$5) \quad 4 \sin x \cdot \cos^3 x - 4 \cos x \cdot \sin^3 x = \sin 4x$$

$$6) \quad \frac{\cos x}{\sin x - 1} - \frac{1}{\left( \tan x - \frac{1}{\cos x \cdot \sin x} \right)} = \frac{-1}{\cos x}$$

*Trigonometry 4, exercise:*

*Prove the following identities:*

$$1) \quad (\tan^2 x + \sin^2 x + \cos^2 x) \frac{1}{\tan^2 x} = \frac{1}{\sin^2 x}$$

$$2) \quad \frac{\frac{1}{\cos(180^\circ + x)} \cdot \tan(90^\circ + x) + \cos(90^\circ - x)}{\sin(180^\circ - x)} = \frac{-1}{\tan^2 x}$$

$$3) \quad \frac{1 - \sin x}{1 + \sin x} = \left( \frac{1}{\cos x} - \tan x \right)^2$$

$$4) \quad \frac{\tan x(1 - \sin^2 x) - \cos^2 x}{\sin^2 x - \cos^2 x} = \frac{\cos x}{\sin x + \cos x}$$

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**Trigonometry 5**  
**General solution**

*Trigonometry 5, practice examples 1:*

Find the general solution for the following:

- |                                   |  |
|-----------------------------------|--|
| 1) $\sin \delta = 0.5$            | 2) $\cos \delta = \frac{-\sqrt{3}}{2}$                             |
| 3) $\tan \delta = 3$              | 4) $-2 \sin \delta = 1.8$  |
| 5) $12 \sin \delta = \cos \delta$ | 6) $5 \tan \delta - \frac{1}{\cos \delta} = \frac{4}{\cos \delta}$ |

*Trigonometry 5, practice examples 2:*

Find the general solution for the following:

- 1)  $\sin 2x = \cos x$
- 2)  $\sin \frac{x}{3} = \cos(60^\circ + x)$
- 3)  $\cos x = \sin(150^\circ - x)$
- 4)  $6 \sin^2 x + 3 \sin x = 3$

*Trigonometry 5, exercise:*

Find the general solution:

- |                                       |  |
|---------------------------------------|--|
| 1) $\cos \delta = \frac{\sqrt{2}}{2}$ | 2) $\sin \delta = \frac{-\sqrt{3}}{2}$     |
| 3) $\tan \delta = 1.35$               | 4) $\cos \frac{x}{3} = \sin(45^\circ + x)$ |
| 5) $3 \sin x = \sin x + 1$            | 6) $2 \tan^2 x - 10 \tan x = -12$          |

**Trigonometry 6**  
**Trigonometric graphs**

*Trigonometry 6, practice examples 1:*

*Sketch the following graphs for the period  $x \in \{-360^\circ; 360^\circ\}$*

- 1) a)  $\sin x$       b)  $\sin 2x$       c)  $2\sin x$       d)  $\sin(x + 30^\circ)$
- 2) a)  $\cos x$       b)  $\cos 2x$       c)  $2\cos x$       d)  $\cos(x + 30^\circ)$
- 3) a)  $\tan x$       b)  $\tan 2x$       c)  $2 \tan x$       d)  $\tan(x + 30^\circ)$

*Trigonometry 6, practice examples 2:*

*Sketch the following on the same set of axis:*

$$f(x) = \sin 2x \quad \text{and} \quad g(x) = \cos(x + 30^\circ)$$

*Trigonometry 6, practice examples 3:*

*Using the graphs  $f(x)$  and  $g(x)$ , determine:*

- 1) Points of intersection
- 2)  $x$  values for which  $f(x) \geq g(x)$
- 3)  $x$  values for which  $f(x) \leq g(x)$
- 4)  $x$  values for which  $f(x)$  is increasing while  $g(x)$  is decreasing

*Trigonometry 6, practice examples 4:*

*On the same set of axis, for the period  $x \in \{-180^\circ; 180^\circ\}$ , sketch the graphs:*

$$f(x) = \sin(x - 45^\circ) \quad \text{and} \quad g(x) = \cos 3x$$

*Using the graphs determine:*

- 1)  $x$  values for which  $f(x) = g(x)$
- 2)  $x$  values for which  $f(x) \geq g(x)$
- 3)  $x$  values for which  $f(x)$  is increasing while  $g(x)$  is decreasing

*Trigonometry 6, exercise:*

*On the same set of axis, for the period  $x \in \{-180^\circ; 180^\circ\}$ , sketch the graphs:*

$$f(x) = \cos(x + 30^\circ) \quad \text{and} \quad g(x) = \sin 2x$$

*Using the graphs determine:*

- 1)  $x$  values for which  $f(x) = g(x)$
- 2)  $x$  values for which  $f(x) \geq g(x)$
- 3)  $x$  values for which  $f(x)$  is increasing while  $g(x)$  is decreasing

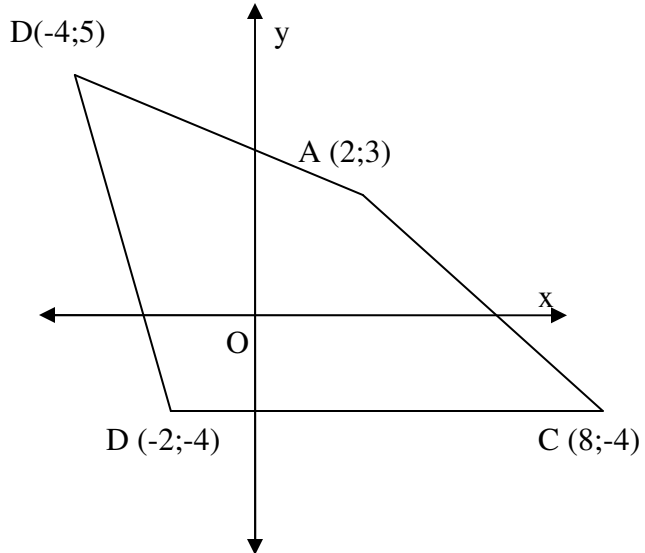
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**Transformations 1**  
**Transformations.**

Transformations 1, practice examples 1:

Sketch the following transformations:

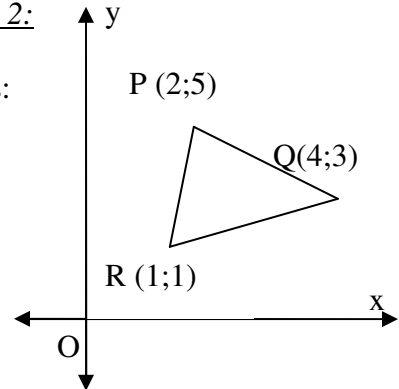
- 1)  $(x ; y) \rightarrow (x ; y+3)$
- 2)  $(x ; y) \rightarrow (x ; y-3)$
- 3)  $(x ; y) \rightarrow (x+2 ; y)$
- 4)  $(x ; y) \rightarrow (x-3 ; y)$
- 5)  $(x ; y) \rightarrow (3x ; 3y)$
- 6)  $(x ; y) \rightarrow \left(\frac{x}{2} ; \frac{y}{2}\right)$



Transformations 1, practice examples 2:

Sketch the following rotations and determine new co-ordinates:

- 1)  $90^\circ$  clockwise
- 2)  $180^\circ$  clockwise
- 3)  $270^\circ$  clockwise



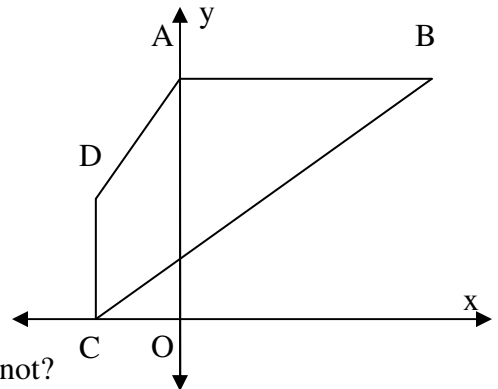
Transformations 1, practice examples 3:

Consider the following sketch with co-ordinates:

A (0 ; 4), B (3 ; 4), C (-1 ; 0) and D (-1 ; 2)

Find co-ordinates of A'B'C'D' if figure:

- 1) rotated  $90^\circ$  clockwise
- 2) rotated  $90^\circ$  anticlockwise
- 3) rotated  $180^\circ$
- 4) rotated  $60^\circ$  clockwise
- 5) rotated  $60^\circ$  anticlockwise
- 6) What are co-ordinates of A''B''C''D'' if figure is rotated  $90^\circ$  clockwise then enlarged by a factor of 2?
- 7) Is the transformation from ABCD to A''B''C''D'' rigid or not?
- 8) Write down the general transformation of ABCD to A''B''C''D''
- 9) What is the ratio of the area of A''B''C''D'' : ABCD



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**Data Handling 1**  
**Basic data analysis**

*Data handling 1, practice examples 1:*

- 1) Of the following set of data, determine the Mean, median and mode and sketch the Box and Whisker plot diagram:

5	8	13	11	7	9	10	8	11	9
6	9	4	10	12	9	8	7	9	6
12	7	5	8	9	10	12	9	10	7

Which measure of central tendency is better?

- 2) The following are running times of learners in a particular class, calculate the mean and variance of the data:

19.5	16.0	15.5	15.1	14.2	14.3	16.0	18.1	18.0	15.3
18.1	14.2	14.2	16.8	16.1	15.0	15.0	16.7	16.9	14.3
13.2	13.0	14.1	14.5	18.1	13.1	16.3	14.6	14.3	15.9

*There is no exercise for this section.*

## MATH DVD WORKBOOK

### EXPONENTS

- 1.1 - <http://www.youtube.com/watch?v=NmyH4I2h9kE>
- 1.2 - <http://www.youtube.com/watch?v=98HYWvJHslg>
- 1.3 - <http://www.youtube.com/watch?v=TLK5jxqqOSY>
- 1.4 - <http://www.youtube.com/watch?v=5Eovopt8AIY>
- 2.1 - <http://www.youtube.com/watch?v=NYIET5zWjw8>
- 2.1 - <http://www.youtube.com/watch?v=ZyOSqHdJh4o>

### ALGEBRA

- 1.1 - <http://www.youtube.com/watch?v=vhWHxgLLkCI>
- 2.1 - <http://www.youtube.com/watch?v=sew84tIKUo8>
- 2.2 - <http://www.youtube.com/watch?v=FkoKapaycaA>
- 2.3 - <http://www.youtube.com/watch?v=FkoKapaycaA>
- 2.4 - <http://www.youtube.com/watch?v=INRpyU4YMKE>
- 3.1 - <http://www.youtube.com/watch?v=W-LWUpiVcXU>
- 3.2 - <http://www.youtube.com/watch?v=KeI03xO8yuo>
- 3.3 - <http://www.youtube.com/watch?v=4E8RCI9CpsI>
- 3.4 - [http://www.youtube.com/watch?v=fpLsuRfc\\_84](http://www.youtube.com/watch?v=fpLsuRfc_84)
- 3.5 - [http://www.youtube.com/watch?v=mK5WhT\\_nI0E](http://www.youtube.com/watch?v=mK5WhT_nI0E)
- 4.1 - <http://www.youtube.com/watch?v=dCpaRaTKh58>
- 4.2 - <http://www.youtube.com/watch?v=W0JPoxPRaSk>
- 4.3 - <http://www.youtube.com/watch?v=-LmWp5HMqjI>

### FUNCTIONS

- 1.1 - [http://www.youtube.com/watch?v=G1\\_w-1akaPk](http://www.youtube.com/watch?v=G1_w-1akaPk)
- 1.2 - <http://www.youtube.com/watch?v=EgeV22oWE3s>
- 1.3 - <http://www.youtube.com/watch?v=NQNWj4JTDNg>
- 1.4 - <http://www.youtube.com/watch?v=l32Sc4EKHGU>
- 2.1 - <http://www.youtube.com/watch?v=YvOXs-rVCAQ>
- 2.2 - <http://www.youtube.com/watch?v=5oEAmEEErws>
- 3.1 - <http://www.youtube.com/watch?v=i2IyRmUi0Nk>
- 3.2 - <http://www.youtube.com/watch?v=ntIVg8Fo4b0>
- 3.3 - <http://www.youtube.com/watch?v=3sgGS7xLwWI>
- 3.4 - <http://www.youtube.com/watch?v=sFmKdjjiispj>
- 3.5 - <http://www.youtube.com/watch?v=WRDcDWLH7Go>
- 3.6 - <http://www.youtube.com/watch?v=168QKqdQmIA>
- 3.7 - [http://www.youtube.com/watch?v=\\_lPhqJ7WDs](http://www.youtube.com/watch?v=_lPhqJ7WDs)
- 3.8 - <http://www.youtube.com/watch?v=-8nyP22DDOs>
- 3.9 - <http://www.youtube.com/watch?v=JRZy3nV8vOg>
- 4.1 - <http://www.youtube.com/watch?v=5b0lDEcNxhw>
- 4.2 - [http://www.youtube.com/watch?v=uVjuux\\_sCck](http://www.youtube.com/watch?v=uVjuux_sCck)
- 4.3 - <http://www.youtube.com/watch?v=Is8YY6pUixw>

## MATH DVD WORKBOOK

- 5.1 - <http://www.youtube.com/watch?v=gnWb0EjSb20>
- 5.2 - <http://www.youtube.com/watch?v=G3xvL0B6ZGI>
- 5.3 - <http://www.youtube.com/watch?v=CwvehjVV4uM>
- 6.1 - <http://www.youtube.com/watch?v=djqxa-IEja8>
- 6.2 - <http://www.youtube.com/watch?v=ZyzGe1lOhyc>

### CALCULUS

- 1.1 - <http://www.youtube.com/watch?v=Op5SZ6AjCD0>
- 1.2 - <http://www.youtube.com/watch?v=kkGgPUq6XxM>
- 1.3 - <http://www.youtube.com/watch?v=s-PZfTNtek4>
- 2.1 - <http://www.youtube.com/watch?v=BhFJPAv6pzM>
- 2.2 - <http://www.youtube.com/watch?v=ozv2RxoXhKA>
- 2.3 - <http://www.youtube.com/watch?v=6Im0xyRyYm8>
- 3.1 - <http://www.youtube.com/watch?v=9uXc1P7ML60>
- 3.2 - <http://www.youtube.com/watch?v=X5ejRYika8s>
- 3.3 - <http://www.youtube.com/watch?v=0aNSZp6XHQw>

### FINANCIAL MATHS

- 1.1 - <http://www.youtube.com/watch?v=4ySlrWmf7p0>
- 1.2 - [http://www.youtube.com/watch?v=dPyriK3Q9\\_k](http://www.youtube.com/watch?v=dPyriK3Q9_k)
- 2.1 - <http://www.youtube.com/watch?v=XaK7mFKs2IU>
- 3.1 - <http://www.youtube.com/watch?v=VfKooQXLTZM>
- 3.2 - <http://www.youtube.com/watch?v=Mfy46Iluy14>